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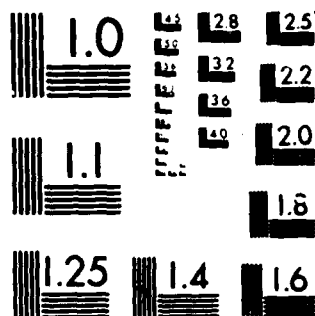
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STATISTICAL SELECTION PROCEDURES

FINAL REPORT ON CONTRACT DAAG-29-81-K-0168

by

Robert E. Bechhofer

Principal Investigator

October 31, 1985

U.S. ARMY RESEARCH OFFICE

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FINAL REPORT ON CONTRACT DAAG-29-81-K-0168

This is the final report on Contract DAAG-29-81-K-0168 entitled "Statistical Selection Procedures." The report covers the period September 1, 1981 through September 15, 1985. This contract with a total budget of \$175,000 was originally planned to cover the 3-year period from September 1, 1981 through August 31, 1984; a no-new-funds extension was granted to continue research through September 15, 1985. The contract is a follow-up to Contract DAAG-29-81-C-0036 (and earlier ones) which had similar missions. Because of the closeness of the objectives of these contracts, the present report lists some articles the research for which was initiated or completed under the earlier contract, but which were not completed or published until after the final report of that contract was submitted.

The following pages describe briefly some of the research results and list the papers published (and those accepted for publication) with the full or partial support of these contracts. The contents of most of these papers have already been described in detail in the eight Semi-Annual Progress Reports covering the 4-year period of performance of the contract.



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## RESEARCH ON CONTRACT DAAG-29-81-K-0168

One of the main thrusts of new research on the present contract stems from a selection problem associated with quantal response curves. The statistical model underlying the quantal response curve problem was described in detail in the Semi-Annual Progress Report on the predecessor contract DAAG-29-80-C-0036 dated June 30, 1980. It was pointed out there that if certain assumptions hold for the  $k \geq 2$  quantal response curves, then the selection problem for the  $k$  quantal response curves can be solved by solving an associated selection problem for  $k \geq 2$  Bernoulli populations. (A follow-up in depth study of the original problem was carried out by Dr. A. C. Tamhane, and reported on in "A survey of literature on estimation methods for quantal response curves with a view toward applying them to the problem of selecting the curve with the smallest  $q$ -quantile (Ed 100q)," TR No. 614 (December 1983); this article has since been accepted for publication in Communications in Statistics. In his article Tamhane assessed the difficulties associated with finding a solution (other than the one mentioned above) to the original problem.)

Recognition of the fact that a solution to the Bernoulli selection problem leads (under appropriate assumptions) to a solution of the quantal response selection problem suggested that intensive study of the former problem would be profitable. Such a study was initiated by the P.I. along with Dr. Radhika Kulkarni, then one of his Ph.D. students. Their study lead to a major research breakthrough; namely an optimal

closed adaptive sequential procedure for selecting the best Bernoulli population. Their findings are not only relevant to the quantile selection problem, but are also very important in their own right-- e.g., in vendor selection and in clinical trials. Subsequent papers by the P.I. and his co-researchers and others studied in depth the properties of the procedure and its performance characteristics. These papers are listed chronologically in the following pages under the heading "References on the Bechhofer-Kulkarni Bernoulli selection procedure." The results have been presented at professional meetings, both theoretical and applied, and were very well received. The P.I. and his co-workers are continuing study of this procedure and several variants of it.

Results obtained on the Bernoulli selection problem have led to related results on the problem of selecting the multinomial event which has the largest probability. Several articles have been published on this subject by the P.I. along with Drs. David Goldsman and Radhika Kulkarni. Research on this problem is continuing.

A third major area of research initiated by the P.I. and Dr. A. C. Tamhane involved the use and construction of a new class of incomplete block designs, which they termed balanced treatment incomplete block (BTIB) designs for comparing several test treatments with a control treatment. Their fundamental paper on this subject was published in Technometrics (1981) (Corrigendum, Technometrics (1982), 24, 171) with follow-up papers in Sankhya (1983) and in Selected Tables in Mathematical Statistics (1985). Their studies of this problem have

led to further related investigations by many other research workers. A partial list of some of the additional articles published on this subject is given in the following pages under the heading "Additional references on balanced treatment incomplete block (BTIB) designs." At the present time the P.I. does not contemplate further studies of this problem.

The P.I. has submitted a research proposal to ARO-D to pursue new investigations in some of the areas described above and in related ones, and to further exploit research findings already obtained. The P.I. also proposed to write a short monograph on Bernoulli selection procedures with special reference to the Bechhofer-Kulkarni closed adaptive sequential procedure. The monograph would be intended for use by practitioners; tables of constants necessary to implement the procedures, and tables describing the performance characteristics of the procedures will be furnished. As presently conceived the monograph will contain theorems but no proofs. The P.I. along with his colleague, Professor Thomas J. Santner of the School of Operations Research and Industrial Engineering at Cornell, also plans to write a monograph on statistical selection procedures. The theme of the book would be alternatives to hypothesis testing--specifically selection and multiple comparisons with emphasis on the former. It would presume a two-semester course in statistical methods through analysis of variance and factorial experiments. The emphasis would be on relevant procedures and their properties but not on proofs. Tables of constants necessary to implement the procedures will be provided.



The year 1984 marked the 30th anniversary of the first published paper on the so-called "indifference-zone" approach to ranking and selection (Bechhofer, R.E. (1984). "A single sample multiple decision procedure for ranking means of normal populations with known variances," Annals of Mathematical Statistics, 25, 16-39). In celebration of this anniversary the P.I. was asked to write an article surveying the progress made to date and highlighting some of the more important papers written during this period. Professors Shanti S. Gupta and S. Panchapakesan have written a companion article with particular reference to the "subset" approach. These will appear in a special issue of the American Journal of Mathematical and Management Sciences. The issue is titled "Statistical ranking and selection: three decades of development."

At this time the P.I. would like to express his thanks to the Army Research Office - Durham for its strong support of this research activity over many years. He is particularly indebted to Dr. Robert Launer of ARO-D for his constant interest and encouragement.

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